The Volume of Antares

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Abstract

The purpose of this article is to elucidate the apparent "contradiction" found in *The Urantia Papers* concerning the volume of the star α Scorpii (Antares). We show that the deviation of the ratio of volumes of two spherical bodies from the cubes of their respective radii (as implied by *The Urantia Papers*) is demanded by the predictions of Einstein's General Relativity. Based on the inequality properties of a star's volume compared to the volume of a disk bounded by a sphere of the same

radius in $\mathbb{R}^3$ we find out the exact definition of "radius" used by the revelators.

1 The Problem

The following statement found in the Section 3 of Paper 41 of *The Urantia Papers* has been a cause of jumping to the conclusion by some prominent members of the general readership that the teachings of this book are in error:

> **The largest star in the universe, the stellar cloud Antares, is 450 times the diameter of your sun and is 60,000,000 times its volume.**

As stated, this obviously contradicts to the formula for the volume of a 3-dimensional disk bounded by a 2-dimensional sphere ($\mathbb{S}^2 = \partial\mathbb{D}^3, \mathbb{D}^3 \subset \mathbb{R}^3$):

$$\frac{V_{Antares}}{V_\odot} = \frac{R^3_{Antares}}{R^3_\odot} = 450^3 \neq 6 \cdot 10^7 \qquad (1)$$

2 The Solution

As long as our description of the reality remains within the conceptual framework of the classical (Newtonian) theory of gravity based on the notion of gravitational field on the scene of a *Galilean spacetime* and the corresponding 3-dimensional *Euclidean space*, the problem cannot be satisfactorily resolved.

We shall now consider the situation from the point of view of the theory of General Relativity formulated by Albert Einstein in 1915. In this theory the gravity is interpreted as the curvature of a Lorentzian spacetime manifold and the "field dynamics" is determined by the metric tensor g_{ik}.

The form of the metric of a centrally-symmetric body is well-known (see e.g. (100.2) in [1]):

$$ds^2 = e^\nu c^2 dt^2 - r^2(d\theta^2 + sin^2\theta\, d\phi^2) - e^\lambda dr^2 \quad (2)$$

where $\nu = \nu(r,t), \lambda = \lambda(r,t)$. The expression (2) is valid both for the *exterior* and *interior* of the body. For the exterior region the exact forms of ν and λ are known as the *Schwarzschild solution* found in 1915:

$$e^{-\lambda} = e^{\nu} = 1 - \frac{2GM}{c^2 r} \tag{3}$$

However, we shall here be only concerned with the *interior* of the star.

A 4-dimensional spacetime metric tensor g_{ik} induces a purely spatial metric on a 3-dimensional space as follows (see e.g. (84.6-7) in [1]):

$$dl^2 = \gamma_{\alpha\beta} dx^{\alpha} dx^{\beta} \tag{4}$$

$$\gamma_{\alpha\beta} = -g_{\alpha\beta} + \frac{g_{0\alpha} g_{0\beta}}{g_{00}} \tag{5}$$

For the specific form of metric (2) we obtain:

$$dl^2 = e^{\lambda} dr^2 + r^2 (d\theta^2 + sin^2\theta \, d\phi^2) \tag{6}$$

From the expression for the spatial metric (6) one can easily ascertain the geometrical meaning of the radial coordinate r as the circumference of a circle with its center at $r = 0$ divided by 2π. Note, that this is *not* the same as the radial spatial distance from the centre. Therefore, we arrive at two non-equivalent definitions of a "radius" of a centrally-symmetric object like a star:

1. As a circumference of any of its great circles divided by 2π.

2. As a distance from the center to the surface, henceforth denoted by R^* to distinguish from the radius according to the previous definited which we shall denote by R.

The expression for dl^2 allows to write the relation between R and R^* straightaway:

$$R^* = \int\limits_0^R e^{\frac{\lambda(r,t)}{2}} dr \tag{7}$$

Certain general considerations (p.302 in [1]) place the following constraints on the function $\lambda(r,t)$:

$$r \to 0 \Rightarrow \lambda/r^2 \to 0 \tag{8}$$

$$\lambda \geqslant 0 \tag{9}$$

The constraint (9) implies that the radius of a star measured as a distance from the centre is always *longer* than the value implied from the circumference of a great circle:

$$R^* \geqslant R \tag{10}$$

The same constraint implies that the volume of a star with the radius R is always *greater* than the volume a 3-dimensional disk bounded by a sphere of the same radius R in Euclidean 3-dimensional space $\mathbb{R}^3$:

$$V = 4\pi \int_0^R r^2 e^{\frac{\lambda}{2}} dr \geqslant 4\pi \int_0^R r^2 dr = \frac{4}{3}\pi R^3 \tag{11}$$

Therefore, the revelators of *The Urantia Papers* could not possibly have used the circumference-derived notion of radius R in their statement. Let us see if it could have been the notion of the distance from the centre R^*

instead. We shall now prove that this must have in fact been the case. We shall need to establish the following inequality:

$$V_R^* \leqslant \frac{4}{3}\pi(R^*)^3 \tag{12}$$

where V_R^* is the volume of the star with the radius R^* as measured from the center. It is convenient to express this volume in terms of the circumference-implied radius R, where R is related to R^* by (7):

$$4\pi \int_0^R r^2 e^{\frac{\lambda}{2}} dr \leqslant \frac{4\pi}{3} \left(\int_0^R e^{\frac{\lambda}{2}} dr \right)^3 \tag{13}$$

Let us define the following function $f(R)$:

$$f(R) = \frac{1}{3} \left(\int_0^R e^{\frac{\lambda}{2}} dr \right)^3 - \int_0^R r^2 e^{\frac{\lambda}{2}} dr \tag{14}$$

Obviously, $f(0) = 0$ and we need to establish that $f(R) \geqslant 0$ for all $R > 0$. Differentiating (14) by R we have:

$$f'(R) = e^{\frac{\lambda(R,t)}{2}} (\int_0^R e^{\frac{\lambda}{2}} dr)^2 - R^2 e^{\frac{\lambda(R,t)}{2}} \qquad (15)$$

or, substituting the expression (7) for R^* we obtain:

$$f'(R) = e^{\frac{\lambda(R,t)}{2}} \{(R^*)^2 - R^2\} \qquad (16)$$

Now, using the already established inequality (10) we arrive at:

$$f'(R) \geqslant 0 \qquad (17)$$

Therefore, the function $f(R)$ is monotonically increasing everywhere and, having the value 0 at $R = 0$ it must be non-negative for all $R > 0$. □

There are only two possible definitions of the radius of a star for the type of spacetime geometryc produced by a centrally-symmetric body and only the "distance from the center" definition (i.e. R^*) satisfies the required condition of the star's volume being *less* than the corresponding volume of a disk bounded by a sphere in $\mathbb{R}^3$, we conclude that this (R^*) is what has in fact been used by the revelators of *The Urantia Papers* in their statement on the volume of Antares. Or, alternatively, the shape of Antares is not perfectly spherical, but spheroidal.

In conclusion, I would like to make the following two remarks:

1. The actual value of the radius of Antares as given by *The Urantia Papers* $(450 R_\odot)$ is at variance with the value $(800 R_\odot)$ measured by the parallax and observed angular diameter.

2. The rather great factor (1.5) of deviation of the star's volume from the Euclidean value it would have in the absence of space distortion cannot be accounted for by the current view of the internal structure of this star. Indeed, being a class M supergiant with the mass of only $15.5 M_\odot$ it has a density far too low to account for such a substantial curvature. Could some other undiscovered

yet form of energy have caused this spacetime distortion or are the astronomical data and models of Antares simply wrong?

3 Acknowledgments

I am grateful to Michael Cuthbertson who asked me to provide more details to my claim on 3^{rd} May 2010 on `truthbook.com` forum where I simply stated (without rigorous proof) that the explanation for the apparent "contradiction" with the volume of Antares is probably due to GR effects. This article is the direct result of his request.

References

[1] L.D. Landau and E.M. Lifshitz. *The Classical Theory of Fields.* Butterworth-Heinemann, Fourth Revised English Edition, 1995.